

Polarization transfer in pulsar magnetosphere

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ABSTRACT

Propagation of radio waves in the ultrarelativistic magnetized electron-positron plasma of pulsar magnetosphere is considered. Polarization state of the original natural waves is found to vary markedly on account of the wave mode coupling and cyclotron absorption. The change is most pronounced when the regions of mode coupling and cyclotron resonance approximately coincide. In cases when the wave mode coupling occurs above and below the resonance region, the resultant polarization appears essentially distinct. The main result of the paper is that in the former case the polarization modes become non-orthogonal. The analytical treatment of the equations of polarization transfer is accompanied by the numerical calculations. The observational consequences of polarization evolution in pulsar plasma are discussed as well.

Key words: waves — plasmas — polarization — pulsars: general

1 INTRODUCTION

1.1 Empirical model of pulsar polarization

The radio emission observed from pulsars is typically characterized by a high percentage of linear polarization. Within the framework of a well-known rotating vector model (Radhakrishnan & Cooke 1969), the orientation of pulsar polarization reflects the magnetic field geometry in the emission region and therefore the position angle (PA) changes monotonically as the pulsar beam rotates with respect to an observer. The characteristic *S*-shaped swing of PA across the pulse is indeed observed in a number of pulsars. In addition, PA may show abrupt jumps by approximately 90° (e.g. Manchester, Taylor & Huguenin 1975), testifying to the presence of the two orthogonally polarized modes (OPMs). The early studies of this phenomenon have revealed that for each of the OPMs PA roughly follows the predictions of the rotating vector model (Backer, Rankin & Campbell 1976) and mode changing is a stochastic process (Cordes, Rankin & Backer 1978). Further on the OPMs have been recognized as a fundamental feature of pulsar radio emission (Backer & Rankin 1980; Stinebring et al. 1984a,b). A comprehensive analysis of the

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observational data has proved an idea of superposed OPMs: At any pulse longitude the radio emission is believed to present an incoherent mixture of the two OPMs, whose intensities vary randomly from pulse to pulse (McKinnon & Stinebring 1998, 2000).

The plasma of pulsar magnetosphere does allow two types of non-damping natural waves, the ordinary and extraordinary ones. They propagate in a superstrong magnetic field, generally at not a small angle to the field lines, and, correspondingly, are linearly polarized in orthogonal directions. The electric vector of the ordinary wave lies in the same plane as the wavevector and the ambient magnetic field, while the extraordinary wave is polarized perpendicularly to this plane. The origin of the two types of natural waves is attributed either to the two distinct radio emission mechanisms (e.g. McKinnon 1997) or to the partial conversion of the ordinary mode into the extraordinary one (Petrova 2001). A recently discovered anticorrelation of the OPM intensities (Edwards & Stappers 2004) favours the latter scenario.

It should be noted, however, that direct identification of the observed superposed OPMs with the natural modes of pulsar plasma faces serious difficulties. First of all, some circular polarization is always present in pulsar radiation. Usually it is much lower than its linear counterpart but not negligible. It has been noticed that the sign of circular polarization is well correlated with PA (Cordes et al. 1978), that is the two OPMs have circular polarization of opposite signs and can be regarded as purely orthogonal elliptical modes. As for the theoretical interpretation, immediate switching to the case of elliptical natural waves (Melrose & Stoneham 1977; Melrose 1979; von Hoensbroech, Lesch & Kunzl 1998; Melrose & Luo 2004a; Luo & Melrose 2004) seems problematic. The ellipticity of the natural waves may result from the gyrotropy of pulsar plasma (caused by difference in the distributions of electrons and positrons), if only the waves propagate quasi-longitudinally with respect to the magnetic field. Although the plasma gyrotropy is very probable, the regime of quasi-longitudinal propagation is not characteristic of pulsar magnetosphere. It can be the case only at some specific locations (most likely, close to the magnetic axis, where the divergence of magnetic field lines is less significant) and cannot account for the elliptically polarized natural waves observed throughout the pulse and over a wide frequency range.

Recent thorough studies of the single-pulse data have introduced further complications into the picture of pulsar polarization. It has been found that the observed fluctuations of the Stokes parameters cannot be explained solely by the pulse-to-pulse variation of the OPM intensities (McKinnon 2004; Edwards & Stappers 2004). To account for the observations it has been suggested to complement the OPMs with a randomly polarized component of unknown nature (McKinnon 2004). Alternatively, the same results can be interpreted as a consequence of pulse-to-pulse jitter of both the ellipticity and PA (Karastergiou, Johnston & Kramer 2003; Edwards & Stappers 2004). However, even this generalized picture based on the OPMs with the randomly varying vector of the Stokes parameters appears incomplete. In some cases, the modes are clearly non-orthogonal (Karastergiou et al. 2003; McKinnon 2004; Edwards & Stappers 2004). It should be mentioned that the non-orthogonality of polarization modes manifests itself not only in PA, but also in the circular polarization. In particular, the same PA may be accompanied by the circular polarization of any sense, the correlation between PA and V being less perfect at higher frequencies (Karastergiou et al. 2001, 2003).

Diverse and complicated behaviour of the single-pulse polarization as well as its strong frequency dependence motivate our study of the propagation effects in pulsar magnetosphere.

1.2 Wave mode coupling in pulsar magnetosphere

Pulsar radio emission is believed to be generated deep inside the tube of open magnetic lines. Then it should propagate through the flow of an ultrarelativistic highly magnetized electron-positron plasma. As a result of propagation effects, polarization of the radio waves may evolve significantly. In the vicinity of the

emission region, the ordinary and extraordinary waves are linearly polarized in orthogonal directions and the plasma number density is so large that the geometrical optics regime holds: the natural waves propagate independently, with the electric vectors being adjusted to the orientation of the ambient magnetic field. As the plasma number density decreases with distance from the neutron star, the difference in the refractive indices of the waves decreases as well, and finally the scale length for beating between the modes becomes comparable to the scale length for change in the plasma parameters. Then the polarization planes of the waves have no time to follow the local magnetic field direction, geometrical optics approximation is broken and wave mode coupling starts. Typically this occurs in the outer magnetosphere, at distances of a few tenth of the light cylinder radius. In the region of wave mode coupling, each of the incident natural waves becomes a coherent sum of both natural waves peculiar to the ambient plasma, with the amplitude ratio and phase difference varying along the trajectory. Correspondingly, the ellipticity of the waves increases with distance and the major axis of polarization ellipse is monotonically shifted, so that it no longer reflects the orientation of the ambient magnetic field. Further on, as the plasma density decreases considerably, the waves decouple from the plasma and propagate just as in vacuum, preserving their elliptical polarization. Therefore the process of wave mode coupling is usually called polarization-limiting effect.

This effect has long been used to explain the origin of circular polarization in pulsar radio emission (Cheng & Ruderman 1979; Radhakrishnan & Rankin 1990; Lyubarskii & Petrova 1999). The numerical tracings of polarization evolution in pulsar plasma have demonstrated that the mode coupling effect is strong enough to have marked observational consequences (Petrova & Lyubarskii 2000; Petrova 2001, 2003a).

Polarization evolution of radio waves in pulsar plasma differs significantly from the evolution in the interstellar medium. In contrast to the case of Faraday rotation, within the pulsar magnetosphere the natural waves propagate quasi-transversely with respect to the magnetic field and have linear polarization. This rather corresponds to the Cotton-Mouton birefringence (or so called generalized Faraday rotation, in terms of the paper by Kennet & Melrose 1998). Another important distinctive feature of polarization evolution in pulsar magnetosphere is that it takes place in an essentially inhomogeneous medium. The magnetic field of a pulsar has approximately dipolar structure, and furthermore, because of continuity of the plasma flow in the tube of open magnetic lines, the plasma number density decreases rapidly with distance. Thus, the character of polarization evolution in pulsar magnetosphere is quite specific, though the underlying physics of birefringence is certainly the same.

As a result of the mode coupling effect, the outgoing waves acquire purely orthogonal elliptical polarization, matching the empirical representation of superposed elliptical OPMs. It is important to note that the degree of circular polarization of the modes and their shift in PA are related to each other, both being determined by the parameters of the plasma flow in the region of wave mode coupling (Petrova 2003a). Hence, the observed pulse-to-pulse variations in the ellipticity and PA of the OPMs can be attributed to the fluctuations in pulsar plasma. Besides that, the propagation origin of pulsar polarization should imply a correlation between the values of the ellipticity and PA of the OPMs at a given pulse longitude. An evidence for such a correlation has recently been found in Edwards (2004) (for more details see Petrova 2006). Thus, the mode coupling effect can account for a number of important features of the observed single-pulse polarization.

At the same time, the question as to the origin of non-orthogonality of polarization modes still remains open. The observational manifestations of this phenomenon have recently been reported in a number of papers (e.g. Edwards & Stappers 2004; McKinnon 2004; Ramachandran et al. 2004; Karastergiou et al. 2003, 2001). In the present paper, we concentrate on a more detailed treatment of polarization evolution in pulsar magnetosphere, which, in particular, explains non-orthogonality of the modes.

1.3 Statement of the problem

Polarization evolution in pulsar magnetosphere has previously been considered in the approximation of a superstrong magnetic field. It means that in the rest frame of the plasma flow the radio frequency, ω' , is much less than the electron gyrofrequency, ω_H . In other words, the radius of cyclotron resonance, where $\omega' = \omega_H$, has been assumed to be infinitely large. At the conditions relevant to pulsar magnetosphere, the radius of cyclotron resonance is often somewhat larger than that of the mode coupling region, but generally these quantities are of the same order of magnitude (Barnard 1986, see also equation (15) below). Therefore it is reasonable to inspect the role of the cyclotron resonance in the evolution of pulsar polarization.

In application to pulsars, the cyclotron absorption has been considered in a number of papers (Blandford & Scharlemann 1976; Lyubarskii & Petrova 1998; Petrova 2002, 2003b) and found efficient, especially in case of small pitch-angles of the absorbing particles. In these papers, it is assumed that the resonant photons interact with a system of absorbing particles rather than with the plasma. The main motivation for such an assumption is that the resonance region lies in the outer magnetosphere, where the plasma number density is small enough. Indeed, in case of pulsars, taking into account the plasma effect on the process of cyclotron absorption introduces only small corrections to the absorption coefficients of the natural modes and does not change the total intensities of outgoing radiation considerably (Lyubarskii & Petrova 1998). At the same time, cyclotron absorption in the plasma may markedly affect polarization evolution of radio waves. The contribution of cyclotron absorption, though not large quantitatively, may appear comparable to that of the mode coupling effect, modifying the final polarization of a pulsar drastically.

Given that the regime of geometrical optics is still valid in the region of cyclotron resonance, the ordinary and extraordinary waves are absorbed independently, with the absorption coefficients being slightly different. Note that this difference is purely the plasma effect, not characteristic of a simple system of absorbing particles. Beyond the resonance region, the waves propagate in the weakly magnetized plasma with rapidly decreasing number density and finally suffer the mode coupling. It should be noted that in case of weakly magnetized plasma the limiting polarization differs substantially from that for the superstrong magnetic field.

Given that the natural waves pass through the coupling region before the cyclotron resonance, in the resonance region each of the waves presents a coherent mixture of the two natural waves. Since for these constituents absorption is not identical, the wave polarization changes considerably. It is important to note that for the two incident waves polarization evolution is not the same, since they contain different portions of the ordinary and extraordinary waves. As a result, polarization states of the outgoing waves are non-orthogonal.

In the present paper, we concentrate on the analytical consideration of polarization transfer in pulsar plasma, taking into account the effect of cyclotron resonance. The plan of the paper is as follows. In Sect. 2, the main equations are derived, which describe the evolution of the wave fields in the inhomogeneous hot plasma embedded in the magnetic field. The basic numerical estimates are also given there. In Sect. 3, we solve the equations of polarization transfer in the two limiting cases, when the resonance region is well below and well above the mode coupling region, respectively. Section 4 contains the results of numerical tracing of polarization evolution. In Sect. 5, the observational consequences of polarization transfer in pulsar magnetosphere are discussed. Section 6 contains a brief summary. A statistical model of single-pulse polarization based on the propagation effects studied will be developed in the forthcoming paper (Petrova 2006).

2 GENERAL THEORY OF POLARIZATION EVOLUTION IN PULSAR MAGNETOSPHERE

2.1 Basic equations

Let radio waves propagate in the ultrarelativistic highly magnetized electron-positron plasma of pulsar magnetosphere. The problem on polarization evolution of the waves in case of infinitely strong magnetic field has been considered in Lyubarskii & Petrova (1999) and Petrova & Lyubarskii (2000). The wave propagation in a cold relativistically streaming plasma embedded in the magnetic field of a finite strength has been studied in Petrova (2001). However, in that paper the plasma number density has been assumed low enough for the polarization evolution to cease well below the radius of cyclotron resonance and the wave passage through the resonance has not been considered. Below we derive the generalized equations which allow for the cyclotron resonance and are applicable to the more realistic case of a hot plasma.

The evolution of the wave fields \mathbf{E} and \mathbf{B} is described by the Maxwell's equations:

$$\nabla \times \mathbf{E} = \frac{i\omega}{c} \mathbf{B}, \quad (1)$$

$$\nabla \times \mathbf{B} = -\frac{i\omega}{c} \mathbf{E} + \frac{4\pi}{c} \sum_{\alpha} \mathbf{j}_{\alpha}. \quad (2)$$

Here \mathbf{j}_{α} is the linearized current density of the electrons or positrons, summation is over the particle species, and the time dependence is taken in the form $e^{-i\omega t}$. We are going to trace polarization evolution starting from the place where refraction is already inefficient and the natural waves propagate straight. Hence, one can choose a three-dimensional Cartesian system with the z -axis along the wave trajectory and the xz -plane being the plane of the field lines of the dipolar magnetic field of a pulsar at the starting point. Then all the quantities in equations (1)-(2) depend only on the z -coordinate.

Since the refractive indices of both natural waves, the ordinary and extraordinary ones, are close to unity, the wave electric field can be presented as

$$E_{x,y,z} = \tilde{E}_{x,y,z} e^{i\frac{\omega}{c}z}, \quad (3)$$

where $\tilde{E}_{x,y,z}$ vary weakly over the wavelength:

$$\frac{d\tilde{E}_{x,y,z}}{dz} \ll \tilde{E}_{x,y,z} \frac{\omega}{c}. \quad (4)$$

All the other quantities entering equations (1)-(2) can be presented similarly. Further on the tildes will be omitted. The representation (3)-(4) corresponds to the case of a weakly inhomogeneous medium. Typically the scale length for change in the plasma parameters is about the altitude above the neutron star, $10^7 - 10^8$ cm, and the assumption of weak inhomogeneity is justified. Note that equation (4) requires also the length of the cyclotron resonance region to be much more than the wavelength. This is valid for the hot plasma, in which case the wave passes successively through the resonance with the particles of different energies.

Substituting equation (1) into equation (2) and making use of equations (3) and (4) yields:

$$\begin{aligned} \frac{dE_x}{dz} + \frac{2\pi}{c} \sum_{\alpha} j_{\alpha x} &= 0, \\ \frac{dE_y}{dz} + \frac{2\pi}{c} \sum_{\alpha} j_{\alpha y} &= 0, \\ E_z + \frac{4\pi i}{\omega} \sum_{\alpha} j_{\alpha z} &= 0. \end{aligned} \quad (5)$$

As can be seen from the above equations, $E_z \ll E_{x,y}$, that is the waves are practically transverse; hereafter it is taken that $E_z = 0$. Furthermore, for the components of \mathbf{j}_α in the first two equations it is sufficient to use the expressions obtained in the approximation of a homogeneous medium, where the fields and currents are $\propto e^{i\frac{\omega}{c}z}$, since the corrections for the medium inhomogeneity are too small (cf. equation (4)). The conductivity tensor of a hot magnetized homogeneous plasma is well known (e.g. Mikhailovsky 1975), however, it is typically written for the coordinate system with the z-axis along the magnetic field direction rather than along the wavevector, as is necessary for our problem. The corresponding transformation of the conductivity tensor is performed in Appendix. Using those results in equation (5), we find finally:

$$\begin{aligned} \frac{dE_x}{dz} + \frac{i\omega}{2c} [Ab_x(E_x b_x + E_y b_y) - BE_x + iGE_y] &= 0, \\ \frac{dE_y}{dz} + \frac{i\omega}{2c} [Ab_y(E_x b_x + E_y b_y) - BE_y - iGE_x] &= 0. \end{aligned} \quad (6)$$

Here $b_{x,y,z}$ are the direction cosines of the ambient magnetic field,

$$\begin{aligned} A &\equiv \sum_{\alpha} \left[\left[\frac{\omega_{p\alpha}^2 f_{\alpha}(\gamma_{\alpha})}{\gamma_{\alpha} \omega_{\alpha}'^2} \frac{\omega_H^2}{\omega_H^2 - \omega_{\alpha}'^2} \right] \right], \\ B &\equiv \sum_{\alpha} \left[\left[\frac{\omega_{p\alpha}^2 f_{\alpha}(\gamma_{\alpha}) \gamma_{\alpha} (1 - \beta_{\alpha} b_z)^2}{\omega_H^2 - \omega_{\alpha}'^2} \right] \right], \\ G &\equiv \sum_{\alpha} \left[\left[\frac{\omega_{p\alpha}^2 f_{\alpha}(\gamma_{\alpha}) (q_{\alpha}/e) (\omega_H/\omega) (\beta_{\alpha} - b_z)}{\omega_H^2 - \omega_{\alpha}'^2} \right] \right], \end{aligned} \quad (7)$$

$\omega_p \equiv \sqrt{\frac{4\pi e^2 n}{m_e}}$ is the plasma frequency, n the number density, $\omega' \equiv \omega\gamma(1 - \beta b_z)$ is the wave frequency in the frame of the plasma particle moving at a speed of $\beta \equiv v/c$, $\gamma \equiv (1 - \beta^2)^{-1/2}$ the Lorentz-factor, $q_{\alpha} = \pm e$, $f(\gamma)$ the distribution function of electrons or positrons with the normalization $\int f(\gamma) d\gamma \equiv 1$, the double square brackets stand for the Landau integral:

$$\left[\left[\frac{F(\gamma)}{\omega_H^2 - \omega_{\alpha}'^2} \right] \right] = \text{v.p.} \int \frac{F(\gamma) d\gamma}{\omega_H^2 - \omega_{\alpha}'^2} + \pi i \int \frac{\delta(\omega_H - \omega') F(\gamma) d\gamma}{\omega_H + \omega'}, \quad (8)$$

the first of the above integrals is taken in the principal value sense.

Equations (6)-(8) describe polarization evolution of the waves in the hot magnetized weakly inhomogeneous plasma allowing for the effect of cyclotron resonance. These equations coincide with equation (12) in Petrova (2001) in case of cold plasma, with the distribution function $f(\gamma) = \delta(\gamma - \gamma_0)$, if only a part of the wave trajectory well below the resonance radius is considered.

It should be noted that equation (6) incorporates the current density in the limit of small pitch-angles, ψ , of the plasma particles (for more detail see Appendix):

$$\frac{\omega'}{\omega_H} \frac{\psi}{\theta} \ll 1, \quad (9)$$

where $\theta \approx \sqrt{b_x^2 + b_y^2}$ is the angle between the wavevector and the ambient magnetic field. In the resonance region, equation (9) means that $\psi \ll \theta$. Since in pulsar magnetosphere the optical depth to cyclotron absorption is typically large (cf. equation (12) below), the absorbing particles can increase their pitch-angles significantly and the approximation $\psi \ll \theta$ can be broken (Lyubarskii & Petrova 1998; Petrova 2002, 2003b). This especially concerns low enough radio frequencies which meet the resonance condition at higher altitudes above the neutron star and thus interact with the particles whose distribution function has already evolved because of absorption of the higher-frequency radio photons. At the same time, absorption by the

particles with $\psi \ll \theta$ is still characteristic of pulsar magnetosphere, and in the present paper we concentrate on this case of small pitch-angles.

2.2 Numerical estimates

As can be seen from the wave equation (6), the terms containing B can be excluded by introducing the substitution $\hat{E}_{x,y} = E_{x,y} e^{i\frac{\omega}{2\pi} \int B dz}$, i.e. these terms act to decrease both field components identically. With the common expression for the total intensity, $I = E_x E_x^* + E_y E_y^*$ (where the asterisk stands for complex conjugation), one can easily recognize the coefficient of cyclotron absorption:

$$\mu = \frac{\omega}{c} \Im B. \quad (10)$$

Using equations (7) and (8), it can be reduced to the form

$$\mu = \frac{2\pi^2 n e^2}{m c \omega} f\left(\frac{\omega_H}{\omega \theta^2/2}\right), \quad (11)$$

which coincides with that known for the coefficient of absorption by the system of particles in vacuum (e.g. equation (4.13) in Lyubarskii & Petrova 1998). Above it is taken into account that the wave propagation is quasi-transverse with respect to the ambient magnetic field, $\theta \ll 1/\gamma$. For the conditions relevant to pulsar magnetosphere, the optical depth to cyclotron absorption, $\Gamma_c = \int \mu dz$, is estimated as follows (e.g. equation (2.8) in Lyubarskii & Petrova 1998):

$$\Gamma_c = \frac{0.4 \kappa_2 B_{\star 12}^{3/5} \sin^{4/5} \vartheta}{(P^3 \nu_9 \gamma_{1.5})^{3/5}}. \quad (12)$$

Here κ is the plasma multiplicity factor, $\kappa_2 \equiv \frac{\kappa}{10^2}$, B_\star the magnetic field strength at the surface of the neutron star, $B_{\star 12} \equiv \frac{B_\star}{10^{12} \text{ G}}$, ϑ the angle between the rotational and magnetic axes of a pulsar, P the pulsar period, ν the radio frequency, $\nu_9 \equiv \frac{\nu}{10^9 \text{ Hz}}$, $\gamma_{1.5} \equiv \frac{\gamma}{10^{1.5}}$. One can see that at low enough frequencies the absorption depth can be significant, especially for the short-period pulsars, $P \sim 0.1$ s. Below we shall no longer be interested in the consequent intensity decrease (equal for both natural waves) and concentrate on examining the normalized Stokes parameters with an eye to tracing polarization evolution of the waves.

In the wave equation (6), the terms containing G allow for the plasma gyrotropy: if the distribution functions of electrons and positrons are identical, $G \equiv 0$. The problem on the plasma motion in the electromagnetic fields of pulsar magnetosphere is very complex and the self-consistent solution has not been found yet. Thus, the question on the net current density in the magnetosphere is still open. The simplest possibility is that the electrons and positrons differ only in the number densities, with the difference being of order of the Goldreich-Julian number density, i.e. $\Delta n/n \sim 1/\kappa$. Then the relative contribution of the plasma gyrotropy to polarization evolution can be estimated as $\Re G / [(b_x^2 + b_y^2) \Re A] \sim \kappa^{-1} (\theta \gamma)^2 \omega' / \omega_H \sim 0.1 \kappa_2^{-1} \theta_{-1}^2 \gamma_{1.5}^2 \omega' / \omega_H$. In the resonance region, it can become significant, at least for the rays of a specific geometry. However, keeping in mind that true form of G is unknown, we leave out the plasma gyrotropy and the resultant ellipticity of the natural waves. Our aim is to study polarization evolution of the linearly polarized natural waves and, in particular, to demonstrate how the ellipticity arises and changes purely on account of wave mode coupling and cyclotron absorption. An opposite approach has recently been developed by Luo & Melrose (2004) and Melrose & Luo (2004a), who investigated the characteristics of the elliptically polarized natural waves ignoring the above mentioned effects of polarization evolution. The propagation of the elliptical natural waves through the region of cyclotron resonance has been considered in Melrose & Luo (2004b).

In the present paper, we neglect the terms B and G in equation (6) and concentrate solely on A . Note that $\Re A$ describes wave mode coupling, while $\Im A$ corresponds to cyclotron absorption. In the resonance region, these two contributions are roughly of the same order.

Taking into account that the magnetic field strength decreases with distance from the neutron star as z^{-3} , one can estimate the radius of cyclotron resonance, where $\omega\gamma\theta^2/2 = \omega_H$, for the particles with some characteristic Lorentz-factor γ as

$$\frac{z_c}{r_L} = 0.55P^{-1}B_{*12}^{1/3}\nu_9^{-1/3}\gamma_{1.5}^{-1/3}\theta_{-1}^{-2/3}. \quad (13)$$

Here $r_L \equiv 5 \cdot 10^9 P$ cm is the light cylinder radius, $\theta_{-1} \equiv \theta/0.1$. The altitude of the mode coupling region, z_p is another basic quantity of the problem considered. Proceeding from the definition of z_p ,

$$\frac{2\omega_p^2}{\gamma^3\omega^2\theta^2} \frac{\omega}{c} z_p = 1$$

(for more detail see, e.g. Petrova 2003a), one can obtain the following estimate:

$$\frac{z_p}{r_L} = 0.18P^{-3/2}\gamma_{1.5}^{-3/2}\kappa_2^{1/2}B_{*12}^{1/2}\nu_9^{-1/2}\theta_{-1}^{-1}. \quad (14)$$

Hence, the altitudes of the regions of pronounced polarization evolution are in the ratio

$$\frac{z_p}{z_c} = 0.33P^{-1/2}\gamma_{1.5}^{-7/6}\theta_{-1}^{-1/3}B_{*12}^{1/6}\nu_9^{-1/6}\kappa_2^{-1/2}. \quad (15)$$

One can see that for the parameters relevant to pulsars this ratio can take the values more and less than unity. These two possibilities imply qualitatively different scenarios of polarization evolution. In the next Section, we dwell on the analytical treatment of the wave equation in the limiting cases $z_p/z_c \gg 1$ and $z_p/z_c \ll 1$, and find out general features of polarization behaviour in these two scenarios.

3 ANALYTICAL TREATMENT OF POLARIZATION EVOLUTION

In terms of the Stokes parameters,

$$I = E_x E_x^* + E_y E_y^*,$$

$$q = E_x E_x^* - E_y E_y^*,$$

$$u = E_x E_y^* + E_y E_x^*,$$

$$v = i(E_x E_y^* - E_y E_x^*),$$

where the asterisk stands for complex conjugation, the wave equations (6) are written as

$$\begin{aligned} \frac{dI}{dz} &= 2A_2 b_x b_y u + A_2(b_x^2 - b_y^2)q + A_2(b_x^2 + b_y^2)I, \\ \frac{dq}{dz} &= 2A_1 b_x b_y v + A_2(b_x^2 - b_y^2)I + A_2(b_x^2 + b_y^2)q, \\ \frac{du}{dz} &= 2A_2 b_x b_y I - A_1(b_x^2 - b_y^2)q + A_2(b_x^2 + b_y^2)u, \\ \frac{dv}{dz} &= -2A_1 b_x b_y q + A_1(b_x^2 - b_y^2)u + A_2(b_x^2 + b_y^2)v. \end{aligned} \quad (16)$$

Here $A_1 \equiv \frac{\omega}{2c}\Re A$, $A_2 \equiv \frac{\omega}{2c}\Im A$ and, in accordance with the above discussion, the terms containing B and G are left aside. The last terms in equations (16) can be excluded using the substitution $\tilde{s}_m =$

$s_m \exp[-\int A_2(b_x^2 + b_y^2)dz]$, $m = 1, \dots, 4$, where s_m stands for the m -th Stokes parameter. Our primary interest is the evolution of the normalized Stokes parameters, so hereafter we consider the quantities \tilde{s}_m and omit the tildes.

Introducing the normalized coordinate $w \equiv z_p/z$, one can present equations (16) in the following form:

$$\begin{aligned}\frac{dI}{dw} &= -wF_2q - 2\mu F_2u, \\ \frac{dq}{dw} &= -wF_2I - 2\mu F_1v, \\ \frac{du}{dw} &= wF_1v - 2\mu F_2I, \\ \frac{dv}{dw} &= -wF_1u + 2\mu F_1q,\end{aligned}\tag{17}$$

where

$$\begin{aligned}F_1 &\equiv \text{v.p.} \int \frac{f(\gamma)d\gamma}{(\gamma/\gamma_0)^3[1 - (\gamma/\gamma_0)^2(z/z_c)^6]}, \\ F_2 &\equiv \frac{\pi}{2} (z/z_c)^6 \gamma_0 f[(z/z_c)^{-3} \gamma_0],\end{aligned}$$

γ_0 is the characteristic Lorentz-factor of the plasma particles with the distribution function $f(\gamma)$ and z_c is the characteristic altitude of the resonance region defined by the condition $\omega_H(z_c) = \omega\gamma_0\theta^2/2$. Due to continuity of the plasma flow in the tube of open magnetic lines, the plasma number density $n \propto B \propto z^{-3}$. The radio waves are emitted in the plane of magnetic lines and hence initially $b_y = 0$. As the waves propagate in the rotating magnetosphere, they leave the plane of magnetic lines, so that b_y slightly increases along the trajectory, $b_y \sim z/r_L$. For the sake of simplicity, in the present paper this angle is assumed to remain much less than the wavevector tilt in the plane of magnetic lines, $b_x: \mu \equiv (b_y/b_x)_{z_p} \ll 1$.

Strictly speaking, the wave equations (6) correspond to the frame corotating with the neutron star; for the observer's frame the corrections for rotational aberration should be included (cf., e.g. Lyubarskii & Petrova 1999). Similarly to b_y , these corrections are also $\sim z/r_L$ and depend on the angle between the rotational and magnetic axes of the pulsar, the observational geometry and pulse longitude. However, here we are interested only in the principal features of polarization evolution, so we do not specify the geometrical parameters and assume that μ already includes the factor of order unity which allows for rotational aberration.

It is convenient to assume that the particle Lorentz-factors lie within a finite interval $[\gamma_1, \gamma_2]$. Then the resonance region is also finite, with the boundaries given by the conditions $\omega_H(z_{c1,2}) = \omega\gamma_{1,2}\theta^2/2$, and the wave trajectory is divided into the three segments lying below, inside and above the resonance region, respectively. Let us consider the polarization transfer in each of these regions separately.

3.1 The region before cyclotron resonance

Below the region of cyclotron resonance, $F_2 \equiv 0$, and the set of equations (17) is markedly simplified. For the natural waves of pulsar plasma, the initial conditions read: $q_0/I_0 = \pm 1$, $u_0 = v_0 = 0$, where the upper and lower signs correspond to the ordinary and extraordinary waves, respectively. It is easy to see that if $\mu \ll 1$, in the region considered the quantities $|u|$ and $|v|$ remain $\sim \mu$, while $|q| \approx 1$. Thus, equation (17) can be reduced to the form

$$\begin{aligned}\frac{du}{dw} &= wF_1v, \\ \frac{dv}{dw} &= -wF_1u + 2\mu F_1.\end{aligned}\tag{18}$$

Substituting the second of the above equations into the first one, one can find the solution:

$$u_I = \pm 2\mu \Im R_I, \quad v_I = \mp 2\mu \Re R_I, \quad (19)$$

where

$$R_I \equiv e^{-i \int^w F_1 w' dw'} \int_w^\infty e^{i \int^{w'} F_1 w dw} F_1 dw'. \quad (20)$$

The above solution differs substantially for the cases $w_1 \gg 1$ and $w_1 \ll 1$, where w_1 is the lower boundary of the resonance region. It is convenient to introduce the variable $x \equiv w/\eta = z_c/z$, which remains of order unity, and estimate the integral in equation (20) at $\eta \gg 1$ and $\eta \ll 1$.

Given that $\eta \gg 1$, we have

$$u_I \approx \pm \frac{2\mu}{\eta x}, \quad v_I \approx \mp \frac{2\mu}{F_1 \eta^3 x^3}, \quad (21)$$

which is a common solution in the regime of geometrical optics. In the opposite limit, $\eta \ll 1$, the waves pass through the region of wave mode coupling. It is reasonable to present the integral (20) as a sum of the two integrals over the intervals $\infty > w > \sqrt{\eta}$ and $\sqrt{\eta} > w > \eta u_1$. The boundary between the intervals, $u_a = 1/\sqrt{\eta}$, $w_a = \sqrt{\eta}$, is chosen so that w is small enough for the wave mode coupling to be efficient only within the first interval and at the same time it is far enough from the resonance region. In the first interval, the solution has the form

$$\begin{aligned} u_{I_a} &\approx \pm \mu \sqrt{\pi} (1 - w^2/2), \\ v_{I_a} &\approx \mp \mu \sqrt{\pi} (1 + w^2/2) \pm 2\mu w. \end{aligned} \quad (22)$$

To the first order in η , for the second interval we have the following solution:

$$\begin{aligned} u_I &= \pm \mu \sqrt{\pi}, \\ v_I &= \mp \mu \sqrt{\pi} \pm 2\mu \eta \int_{1/\sqrt{\eta}}^{u_1} F_1 du \pm 2\mu \sqrt{\eta}. \end{aligned} \quad (23)$$

Comparing the solutions (21) and (23) obtained for the limiting cases $\eta \gg 1$ and $\eta \ll 1$, respectively, one can see that the waves coming to the resonance region can have substantially distinct polarization characteristics.

3.2 The resonance region

In the resonance region, the set of equations (17) has the form

$$\begin{aligned} \frac{dI}{dx} &= -\eta^2 x F_2 q, \\ \frac{dq}{dx} &= -\eta^2 x F_2 I, \\ \frac{du}{dx} &= \eta^2 x F_1 v - 2\mu \eta F_2 I, \\ \frac{dv}{dx} &= -\eta^2 x F_1 u + 2\mu \eta F_1 q, \end{aligned} \quad (24)$$

where the terms $\sim \mu^2$ are omitted. For the first two equations, one can find the following solutions:

$$q_o = I_o = \exp \left[\eta^2 \int_x^{x_1} F_2 x' dx' \right],$$

$$q_e = -I_e = -\exp\left[-\eta^2 \int_x^{x_1} F_2 x' dx'\right], \quad (25)$$

where the subscripts "o" and "e" correspond to the original ordinary and extraordinary waves, respectively. One can see that the two types of waves evolve in a different manner. It should be kept in mind, however, that actually the ordinary mode intensity does not increase, since it contains an additional factor $\exp[-\eta^2 \int_x^{x_1} F_2 x' dx']$ omitted after equation (16). Thus, the ordinary wave is absorbed by the plasma just as by the system of particles in vacuum (the absorption coefficient is given by equation (11)), whereas the extraordinary wave is absorbed somewhat more efficiently.

Using equations (25) in the last two equations of the set (24), one can obtain:

$$\begin{aligned} v_{II} &= \pm \frac{2\mu}{\eta} \Im R_{II} + v_0 \cos\left(\eta^2 \int_{x_1}^x F_1 x' dx'\right) - \left[u_0 \mp \frac{2\mu}{\eta x_1}\right] \sin\left(\eta^2 \int_{x_1}^x F_1 x' dx'\right), \\ u_{II} &= \pm \frac{2\mu}{\eta} \Re R_{II} \pm \frac{2\mu}{\eta x} \exp\left[\pm \eta^2 \int_x^{x_1} F_2 x' dx'\right] + v_0 \sin\left(\eta^2 \int_{x_1}^x F_1 x' dx'\right) + \left[u_0 \mp \frac{2\mu}{\eta x_1}\right] \cos\left(\eta^2 \int_{x_1}^x F_1 x' dx'\right), \end{aligned} \quad (26)$$

where

$$R_{II} \equiv \exp\left[-i\eta^2 \int_x^{x_1} F_1 x' dx'\right] \times \int_{x_1}^x \frac{\exp\left[i\eta^2 \int_{x'}^{x_1} F_1 x dx\right] \exp\left[\mp \eta^2 \int_{x_1}^{x'} F_2 x dx\right]}{x'^2} dx',$$

$$v_0 \equiv v_I(x_1), \quad u_0 \equiv u_I(x_1).$$

In the limit $\eta \gg 1$, we are interested only in the evolution of the ordinary wave, since the extraordinary one is severely absorbed. The asymptotic treatment of equation (26) yields:

$$(u/I)_{II} = \frac{2\mu}{\eta x}, \quad (v/I)_{II} = -\frac{2\mu}{F_1 \eta^3 x^3}. \quad (27)$$

This solution still coincides with that known for the regime of geometrical optics in the infinitely strong magnetic field (cf. equation (21)). Thus, if in the resonance region the wave propagation obeys geometrical optics, i.e. if $\eta \gg 1$, the cyclotron absorption does not affect polarization state of the wave and acts only to suppress the total intensity.

In the limit $\eta \ll 1$, equation (26) is reduced to the following form:

$$\begin{aligned} u_{II} &= \pm \mu \sqrt{\pi} - 2\mu \eta \int_{x_1}^x F_2 dx', \\ v_{II} &= \mp \mu \sqrt{\pi} \pm 2\mu \eta \int_{1/\sqrt{\eta}}^x F_1 dx' \pm 2\mu \sqrt{\eta}. \end{aligned} \quad (28)$$

In contrast to the case of geometrical optics, in case $\eta \ll 1$ the cyclotron absorption changes the polarization state of the waves (cf. u_I and u_{II}). This can be understood as follows. The waves are subject to wave mode coupling well before the resonance region and become a coherent mixture of the two natural waves. Since these constituents are absorbed not identically, the resultant polarization is modified. For the original ordinary and extraordinary waves, the amplitude ratio of the entering natural waves is essentially distinct, so they suffer different polarization evolution (see equation (28)).

3.3 The region beyond cyclotron resonance

As soon as the waves go out of the resonance region, their intensities stop changing, whereas the polarization still evolves. The final polarization state of the waves at infinity is as follows:

$$\begin{aligned}
u_f &= 2\mu\eta\Im R_{\text{III}} + u_0 \cos \left[\eta^2 \int_0^{x_2} F_1 x dx \right] - v_0 \sin \left[\eta^2 \int_0^{x_2} F_1 x dx \right], \\
v_f &= -2\mu\eta\Re R_{\text{III}} + u_0 \sin \left[\eta^2 \int_0^{x_2} F_1 x dx \right] + v_0 \cos \left[\eta^2 \int_0^{x_2} F_1 x dx \right],
\end{aligned} \tag{29}$$

where

$$R_{\text{III}} \equiv e^{-i\eta^2 \int_0^{x_2} F_1 x dx} \int_0^{x_2} e^{i\eta^2 \int_0^x F_1 x' dx'} F_1 dx,$$

$$u_0 \equiv u_{\text{II}}(x_2), \quad v_0 \equiv v_{\text{II}}(x_2).$$

In case $\eta \ll 1$, equation (29) is reduced to

$$\begin{aligned}
u_f &= u_0 = \pm \mu \sqrt{\pi} - 2\mu\eta \int_{x_1}^{x_2} F_2 dx, \\
v_f &= \mp \mu \sqrt{\pi} \pm 2\mu\eta \int_{1/\sqrt{\eta}}^0 F_1 dx \pm 2\mu\sqrt{\eta}.
\end{aligned} \tag{30}$$

Hence, the wave polarization does not suffer marked changes beyond the resonance region.

Given that $\eta \gg 1$, in the region under consideration, the wave polarization evolves drastically because of wave mode coupling. In equation (29), the addends containing u_0 and v_0 compensate for the first terms of the expansion of R_{III} in power of η^{-2} , and the main contribution comes from the stationary point of R_{III} . The final polarization takes the form

$$\begin{aligned}
(u/I)_f &= 2^{5/8} \mu \eta^{-3/4} \Gamma(7/8) \cos(\pi/16), \\
(v/I)_f &= 2^{5/8} \mu \eta^{-3/4} \Gamma(7/8) \sin(\pi/16),
\end{aligned} \tag{31}$$

where $\Gamma(\cdot)$ is the gamma-function and it is taken into account that far from the resonance region $F_1 \approx -u^6$, independently of the detailed form of the distribution function $f(\gamma)$.

Equations (30) and (31) describe the final polarization of the waves in cases when the region of wave mode coupling lies much lower and much higher than the region of cyclotron resonance, respectively. It should be noted that in the weak magnetic field polarization evolution as a result of wave mode coupling differs essentially from that in the superstrong magnetic field. First of all, the resultant circular polarization has opposite signs (cf. equations (30) and (31)). Besides that, in the weak field, the evolution is less pronounced (note the factors $\eta^{-3/4}$ in equation (31)).

4 NUMERICAL SIMULATION

Now let us turn to numerical simulation of polarization transfer in pulsar plasma based on equation (16). First of all, we introduce a simplified distribution function of the plasma particles in the form of a triangle:

$$f(\gamma) = \begin{cases} \frac{\gamma - \gamma_0(1-a)}{a^2 \gamma_0^2}, & \gamma_0(1-a) \leq \gamma \leq \gamma_0, \\ \frac{\gamma_0(1+a) - \gamma}{a^2 \gamma_0^2}, & \gamma_0 \leq \gamma \leq \gamma_0(1+a). \end{cases} \tag{32}$$

Figure 1 shows the functions $F_1(z_c/z)$ and $F_2(z_c/z)$, which incorporate equation (32), for the two values of the width of $f(\gamma)$: $a = 0.1$ and $a = 0.5$.

According to the approximate solutions (30) and (31), polarization evolution should be most pronounced at $\eta \approx 1$. In Fig. 2, we present the numerically simulated evolution of the normalized Stokes

parameters at $\eta = 1$. One can see that the changes are indeed marked and are not identical for the two types of waves.

The final values of the normalized Stokes parameters u/I and v/I as functions of η at different μ are plotted in Figs. 3a and b, respectively. The analytical approximations given by equations (30) and (31) are shown in asterisks (where appropriate). The polarization parameters orthogonal to those of the original ordinary mode are shown by dotted lines for comparison. As can be seen from Fig. 3, at $\eta \sim 1$ the modes are markedly non-orthogonal. Note also the switch of the sign of v at $\eta \approx 1$.

At large enough η , the extraordinary mode abruptly turns into the ordinary one. This can be understood as follows. Generally speaking, the natural modes are completely independent only infinitely far from the region of wave mode coupling, whereas for finite η in the resonance region each of the waves contains already a small admixture of the other mode. For large enough η , the extraordinary component is absorbed much more efficiently than the ordinary one, so that even in the original extraordinary wave only the ordinary component survives. Note, however, that this regime can hardly be observed, because the ordinary wave intensity is also strongly suppressed by cyclotron absorption.

5 DISCUSSION

As is found above, polarization evolution of radio waves in pulsar magnetosphere can be substantial. The wave polarization varies because of wave mode coupling and cyclotron absorption, the change being most pronounced if the regions of the coupling and resonance approximately coincide.

In cases when the resonance region lies well below and well above the region of wave mode coupling ($\eta \gg 1$ and $\eta \ll 1$, respectively), the resultant polarization is essentially distinct. First of all, this distinction is caused by the different character of the wave mode coupling in the approximations of the weak and strong magnetic field. In the former case, polarization evolution is weaker and the sign of circular polarization is different.

For $\eta \gg 1$ and $\eta \ll 1$, the consequences of cyclotron absorption are also distinct. Given that $\eta \gg 1$, the waves passing through the resonance region still obey the geometrical optics approximation and propagate independently. Then the cyclotron absorption affects only their intensities. In contrast to the cyclotron absorption by a system of particles, in the plasma the wave intensities are suppressed not identically: the extraordinary wave is absorbed more efficiently, the difference strongly increasing with η . The numerical calculation shows that even at $\eta = 1$ ($\mu = 0.3$, $a = 0.1$) the outgoing intensities differ by a factor of about 4.

In case $\eta \ll 1$, the cyclotron absorption acts mainly to change the wave polarization. The waves entering the resonance region present already a coherent mixture of the two types of natural waves, and since these constituents are absorbed not identically, the resultant polarization changes. For the original ordinary and extraordinary waves, polarization evolution on account of cyclotron absorption is distinct, so that they become non-orthogonal.

The features of polarization transfer listed above are believed to have a number of observational consequences. For example, the pulse-to-pulse fluctuations of η around $\eta \approx 1$ can cause the sense reversals of the circular polarization at a given pulse longitude (see also Melrose & Luo 2004b), which are really observed (e.g. Karastergiou et al. 2001). It should be kept in mind, however, that this requires too large fluctuations in the plasma parameters; besides that, on both sides of $\eta = 1$, the absolute values of v are markedly different, while very often the observed sense reversals do not change the absolute value of the circular polarization essentially.

Generally speaking, pulsar radiation presents a mixture of the two types of waves with nearly orthogonal polarizations and comparable randomly varying intensities, so that it is partially depolarized. The cyclotron

absorption, especially at large enough η , introduces a systematic difference in the randomly varying mode intensities. Then the ordinary wave should prevail on average, in which case the degree of polarization should be markedly enhanced. Large η do not seem typical of the pulsars observed, however, some pulsars with exceptionally high polarization indeed exist. The Vela presents a well-known example of such a pulsar. As a rule, in these cases the same polarization mode dominates throughout the average pulse and in most of the single pulses. It is interesting to note that a few strongly polarized pulsars have the total-intensity profiles classified as partial cones (e.g. Lyne & Manchester 1988), which is an additional hint at a significant cyclotron absorption in these objects.

Unfortunately, the actual value of the position angle of linear polarization is not determined in observations and remains unknown, so that direct observations tell nothing as to whether the ordinary or extraordinary mode dominates in the strongly polarized pulsars. Even in case of the Vela, where the X-ray observations of the wind nebula give some new insights into the pulsar geometry, this question has no firm answer. As for the theoretical predictions, cyclotron absorption strongly favours the dominance of the ordinary waves. Let us note in passing that in case of resonant absorption by the particles with large pitch-angles, which is a very probable regime for the Vela, the ordinary waves should also prevail (Petrova 2002, 2003b).

As can be seen from equation (15), η is a very weak function of radio frequency. Obviously, just for this reason the majority of pulsars are detectable over a wide frequency range. Let us speculate, however, that if at the intermediate frequencies $\eta \sim 1$, at low frequencies η is larger and the degree of polarization can substantially increase, which is in line with the observational results below 100 MHz (Suleymanova & Pugachev 2000). At high frequencies $\eta < 1$ and the polarization modes can be markedly non-orthogonal, in which case the degree of circular polarization can be enhanced. The increase of circular polarization at high frequencies is indeed found in the average profiles of a number of pulsars (e.g. von Hoensbroech & Lesch 1999).

A possible non-orthogonality of polarization modes on account of cyclotron absorption has a number of important implications for the statistical model of the individual-pulse polarization of pulsars. (This issue will be considered in detail in the forthcoming paper Petrova 2006). Given that the observed superposed orthogonal polarization modes are associated with the natural waves of pulsar plasma, they should be orthogonal by definition. The process of wave mode coupling also preserves the orthogonality of the waves. To the best of our knowledge, cyclotron absorption is the only way for the *superposed* modes to become non-orthogonal. Note, however, that the non-orthogonal states of the observed *sum* of the modes in different single pulses can be attributed to the time-dependent action of propagation effects.

6 CONCLUSIONS

We have studied polarization transfer in the hot magnetized plasma of pulsars. In the present consideration, we have assumed the non-gyrotropic plasma, with the identical distributions of the electrons and positrons, and the small pitch-angles of the particles. Proceeding from the Maxwell's equations, we have derived the set of equations describing the evolution of the Stokes parameters of the original linearly polarized natural waves. These equations have been solved analytically and the results have been confirmed by numerical calculations.

The polarization evolution of the waves has been found significant. The polarization characteristics change on account of the wave mode coupling and cyclotron absorption. In cases when the region of coupling lies well above and well below the resonance region, the resultant polarization is qualitatively distinct. If the waves pass through the region of wave mode coupling first, they acquire the elliptical polarizations purely orthogonal at the Poincare sphere. Further on, in the resonance region, they become non-orthogonal. If the waves enter the resonance region before coupling, cyclotron absorption does not affect their polarization

states and suppresses the total intensities only, the extraordinary wave being absorbed somewhat more efficiently. Further on the waves suffer mode coupling in the limit of weak magnetic field, which differs from that in the strong field. Firstly, the total change of polarization parameters is much less. Besides that, the resultant circular polarization has the opposite sense.

The observational consequences of polarization transfer in pulsar magnetosphere can be summarized as follows. Because of cyclotron absorption, at large enough η one mode can markedly dominate another one, in which case one can expect strongly polarized profiles, with the same mode dominating throughout the average pulse and in most of the individual pulses. This is indeed characteristic of several pulsars. In some cases, the total-intensity profiles of the strongly polarized pulsars are classified as partial cones, with one of the conal components being absent. This seems to be an additional argument in favour of strong cyclotron absorption in these pulsars. At present it is not known exactly what of the natural waves actually dominates in the strongly polarized pulsars. Our consideration favours the dominance of the ordinary mode.

If one consider η as a function of radio frequency, with $\eta \approx 1$ at the intermediate frequencies, in the low-frequency range $\eta \geq 1$ and one can expect strong polarization of pulsar profiles, whereas at high enough frequencies there may be an increase of the degree of circular polarization because of non-orthogonality of the modes. The non-orthogonality of the outgoing waves is of a crucial importance for the statistical model of the individual-pulse polarization. This will be studied in detail in the forthcoming paper.

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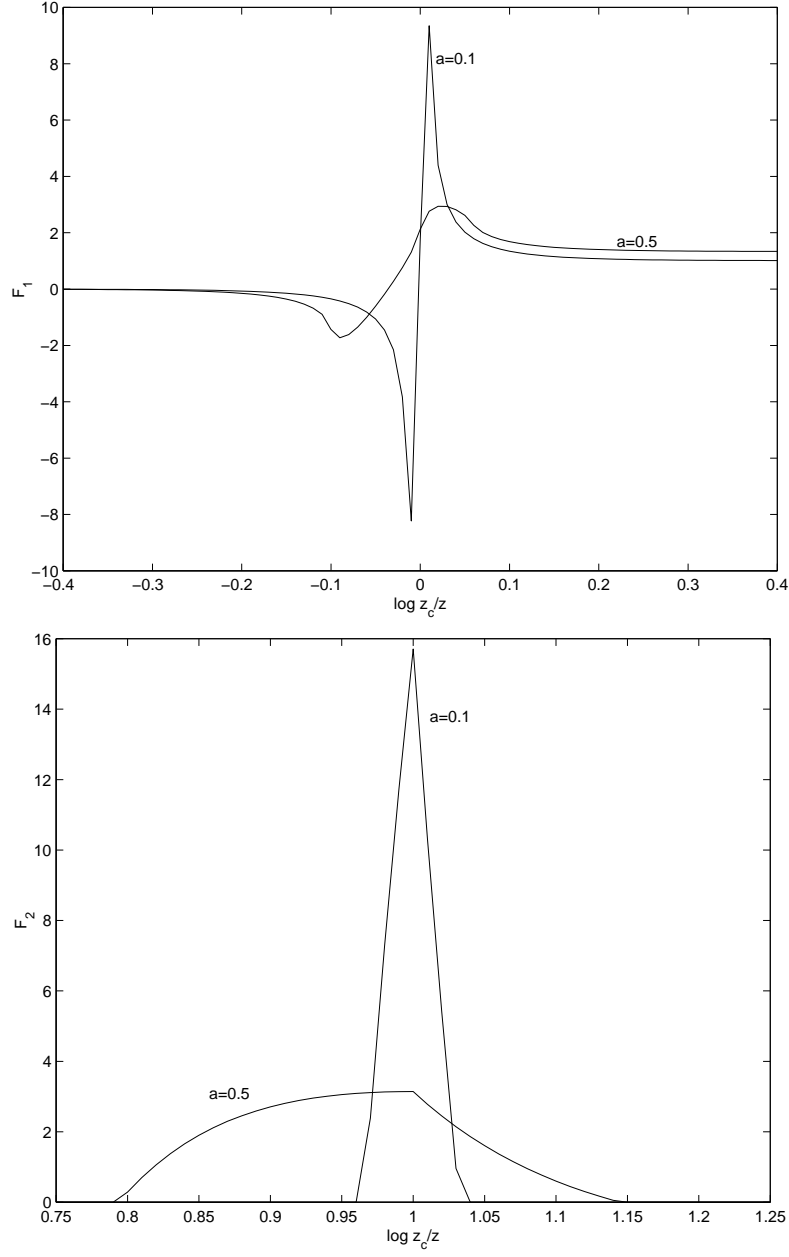
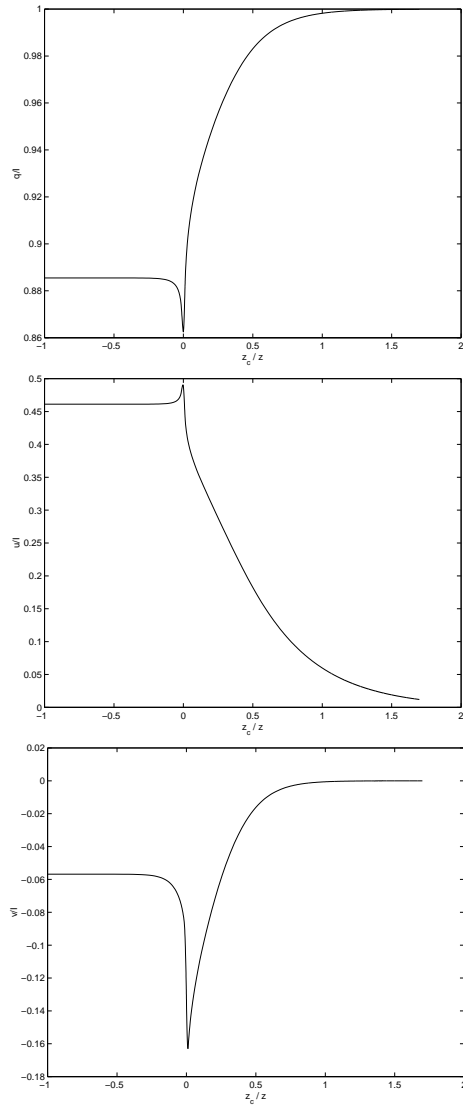


Figure 1. F_1 and F_2 as functions of distance along the wave trajectory given the particle distribution function in the form (32)



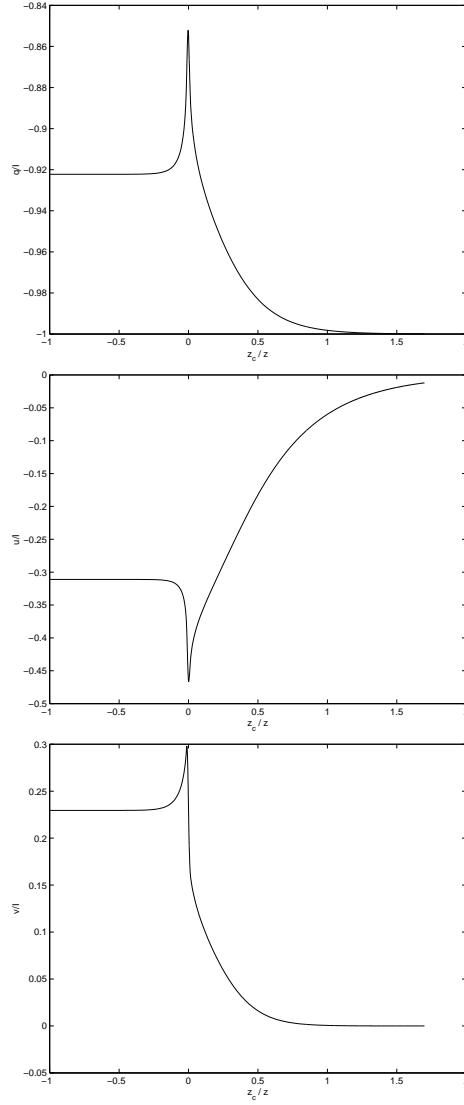


Figure 2. Evolution of the normalized Stokes parameters of the original ordinary (**a-c**) and extraordinary (**d-f**) waves along the trajectory; $\eta = 1$, $\mu = 0.3$, $a = 0.1$.

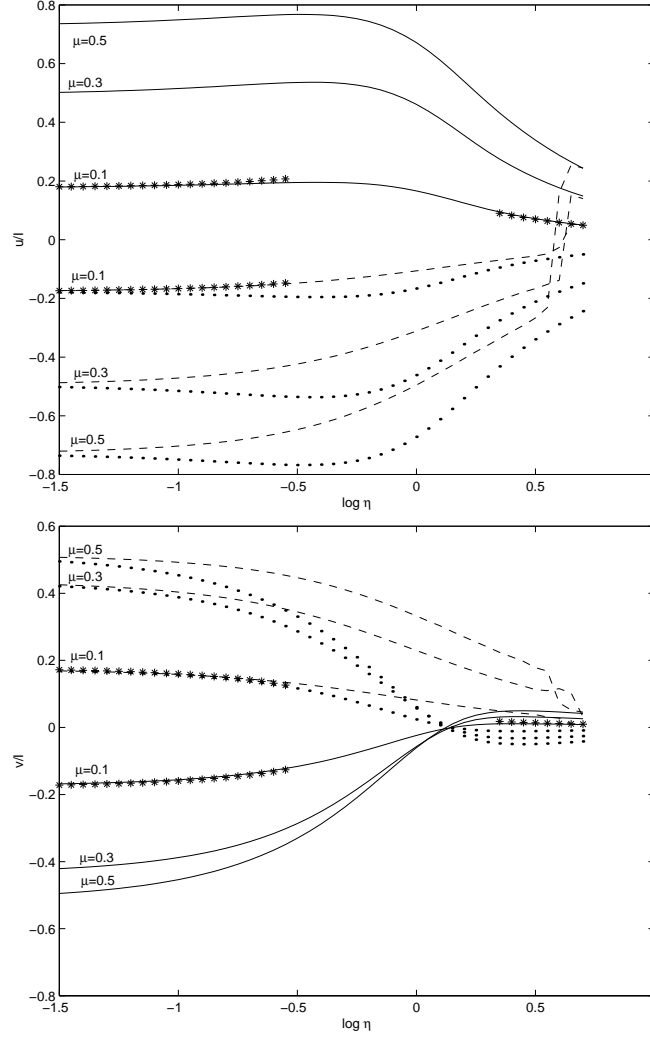


Figure 3. The final normalized Stokes parameters as functions of η for different μ . The analytical approximations given by equations (30) and (31) are shown in asterisks. The solid and dashed lines correspond to the original ordinary and extraordinary waves, respectively. The dotted lines show the parameters orthogonal to those of the ordinary mode.

APPENDIX A: CONDUCTIVITY OF THE HOT MAGNETIZED PLASMA

Our aim here is to find the conductivity tensor of the hot magnetized plasma in the coordinate system with the z-axis along the wavevector. In the commonly used system with the z-axis along the magnetic field direction, it has the following form (e.g. Mikhailovsky 1975):

$$\hat{\sigma}_0 = -ie^2 \times \left\langle \sum_{n=-\infty}^{\infty} \zeta_n \Phi_{\perp} \begin{pmatrix} v_{\perp}^2 \frac{n^2}{\xi^2} J_n^2 & iv_{\perp}^2 \frac{n J_n J'_n}{\xi} & v_{\perp} v_z \frac{n J_n^2}{\xi} \\ -iv_{\perp}^2 \frac{n J_n J'_n}{\xi} & v_{\perp}^2 J_n'^2 & -iv_{\perp} v_z J_n J'_n \\ v_{\perp} v_z \frac{n J_n^2}{\xi} & iv_{\perp} v_z J_n J'_n & v_z J_n^2 \Phi_{\parallel} / \Phi_{\perp} \end{pmatrix} \right\rangle. \quad (\text{A1})$$

Here the brackets $\langle \dots \rangle$ stand for the integral operator $\int \dots p_{\perp} dp_{\perp} dp_z$; J_n and J'_n are the Bessel function of the n -th order and its derivative, ξ is their argument,

$$\xi = k_{\perp} v_{\perp} \gamma / \omega_H;$$

$$\zeta_n = (\omega - k_z v_z - n \omega_H / \gamma)^{-1};$$

$$\Phi_{\perp} = \frac{1}{v_{\perp}} \frac{\partial F}{\partial p_{\perp}} + \frac{k_z}{\omega} \left(\frac{\partial F}{\partial p_z} - \frac{v_z}{v_{\perp}} \frac{\partial F}{\partial p_{\perp}} \right);$$

$$\Phi_{\parallel} = \frac{\partial F}{\partial p_z} - \frac{n \omega_H}{\gamma \omega} \left(\frac{\partial F}{\partial p_z} - \frac{v_z}{v_{\perp}} \frac{\partial F}{\partial p_{\perp}} \right);$$

F is the particle distribution function with the normalization

$$\int F(p_{\perp}, p_z) p_{\perp} dp_{\perp} dp_z = N_0,$$

and the subscripts "z" and " \perp " refer to the components parallel and perpendicular to the magnetic field, respectively.

In terms of the quantities in the system with $Oz \parallel \mathbf{k}$, $v_z = v_{\parallel}$, $k_z = kb_z$, $k_{\perp} = kb_{\perp}$, $b_{\perp} = \sqrt{b_x^2 + b_y^2}$. Then the argument of the Bessel function is written as

$$\xi = \frac{kb_{\perp} v_{\perp} \gamma}{\omega_H} = \frac{\omega'}{\omega_H} \frac{\psi}{b_{\perp}},$$

where $\omega' = \omega \gamma (b_x^2 + b_y^2)$ is the frequency in the rest frame of the particle, ψ the particle pitch-angle and it is taken into account that $kv/\omega \approx 1$, i.e. $v \approx c$ and the refractive index is approximately unity. Hereafter it is assumed that the particle pitch-angle is small, $\psi \ll b_{\perp}$, in which case $\xi \ll 1$ up to the cyclotron resonance, $\omega' = \omega_H$. As can be seen from the final equations (6), beyond the cyclotron resonance, at $\omega' > \omega_H$, the evolution of the electric field amplitudes rapidly ceases because of strong decrease of ω_H , $\omega_H \propto r^{-3}$. Therefore in the case of interest $\xi \ll 1$ and one can use the approximation of the Bessel function:

$$J_n \approx \frac{(\xi/2)^n}{n!}.$$

Then the form of the conductivity tensor is substantially simplified:

$$\sigma_{011} = \sigma_{022} = -ie^2 \left\langle \frac{\gamma \omega (1 - \beta b_z)^2}{m (\omega_H^2 - \omega'^2)} \right\rangle,$$

$$\sigma_{012} = -\sigma_{021} = e^2 \left\langle \frac{\omega_H (1 - \beta b_z)}{m (\omega_H^2 - \omega'^2)} \right\rangle,$$

$$\sigma_{013} = \sigma_{031} = -ie^2 \left\langle \frac{\gamma \omega b_{\perp} \beta (1 - \beta b_z)}{m (\omega_H^2 - \omega'^2)} \right\rangle, \quad (\text{A2})$$

$$\sigma_{023} = \sigma_{032} = -e^2 \left\langle \frac{\omega_H b_{\perp} \beta}{m (\omega_H^2 - \omega'^2)} \right\rangle,$$

$$\sigma_{033} = -ie^2 \left\langle \frac{\gamma \omega b_{\perp}^2 \beta^2}{m (\omega_H^2 - \omega'^2)} - \frac{\omega}{m \gamma \omega'^2} \right\rangle,$$

where the brackets stand for the integral operator $\int \dots F(p_{\parallel}) dp_{\parallel}$.

In the system with $Oz \parallel \mathbf{k}$, the conductivity tensor is given by

$$\hat{\sigma} = \hat{R} \hat{\sigma}_0 \hat{R}^{-1}, \quad (\text{A3})$$

where \hat{R} and \hat{R}^{-1} are the matrices of the direct and inverse transformation between the coordinate systems considered:

$$\hat{R} = \begin{pmatrix} -\frac{b_x b_z}{b_{\perp}} & \frac{b_y}{b_{\perp}} & b_x \\ -\frac{b_y b_z}{b_{\perp}} & -\frac{b_x}{b_{\perp}} & b_y \\ b_{\perp} & 0 & b_z \end{pmatrix}, \quad (\text{A4})$$

$$\hat{R}^{-1} = \begin{pmatrix} -\frac{b_x b_z}{b_{\perp}} & -\frac{b_y b_z}{b_{\perp}} & b_{\perp} \\ \frac{b_y}{b_{\perp}} & -\frac{b_x}{b_{\perp}} & 0 \\ b_x & b_y & b_z \end{pmatrix}. \quad (\text{A5})$$

As is discussed after equation (5), the waves are almost transverse and we take $E_z = 0$. Then we should find only

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y \quad \text{and} \quad j_y = \sigma_{yx} E_x + \sigma_{yy} E_y. \quad (\text{A6})$$

Substituting equations (A4) and (A5) in equation (A3) and taking into account that $\sigma_{011} = \sigma_{022}$, $\sigma_{012} = -\sigma_{021}$, $\sigma_{013} = \sigma_{031}$ and $\sigma_{023} = -\sigma_{032}$, one obtains:

$$\begin{aligned} \sigma_{xx} &= (1 - b_x^2) \sigma_{011} + b_x^2 \sigma_{033} - \frac{2b_x^2 b_z}{b_{\perp}} \sigma_{013}, \\ \sigma_{yy} &= (1 - b_y^2) \sigma_{011} + b_y^2 \sigma_{033} - \frac{2b_y^2 b_z}{b_{\perp}} \sigma_{013}, \\ \sigma_{xy} &= b_x b_y (-\sigma_{011} + \sigma_{033} - 2(b_z/b_{\perp}) \sigma_{013}) + b_z \sigma_{012} + b_{\perp} \sigma_{023}, \\ \sigma_{yx} &= b_x b_y (-\sigma_{011} + \sigma_{033} - 2(b_z/b_{\perp}) \sigma_{013}) - b_z \sigma_{012} - b_{\perp} \sigma_{023}, \end{aligned} \quad (\text{A7})$$

Using equation (A2) in equation (A7) yields finally:

$$\begin{aligned} \sigma_{xx} &= a_1 b_x^2 - a_2, \quad \sigma_{yy} = a_1 b_y^2 - a_2, \\ \sigma_{xy} &= a_1 b_x b_y + g, \quad \sigma_{yx} = a_1 b_x b_y - g, \end{aligned} \quad (\text{A8})$$

with

$$\begin{aligned} a_1 &= \left\langle \frac{ie^2 \omega \omega_H^2}{m \omega'^2 \gamma (\omega_H^2 - \omega'^2)} \right\rangle, \\ a_2 &= \left\langle \frac{-ie^2 \gamma \omega (1 - \beta b_z)^2}{m (\omega_H^2 - \omega'^2)} \right\rangle, \end{aligned}$$

$$g = \left\langle \frac{e^2 \omega_H (b_z - \beta)}{m (\omega_H^2 - \omega'^2)} \right\rangle .$$

Throughout this section, we have considered the one-component plasma for simplicity. The results can be extended to the case of pair plasma by introducing the summation over the particle species.